Truth and Paradox

The concept of truth has been intensively studied by logicians during the twentieth century. In fact, the vigorous attempts that logicians and mathematicians have made to clarify the notion of truth have led to some of the greatest and most far-reaching mathematical discoveries of this century. Much of this work has been inspired by the realization that the very notion of truth seems to give rise to troublesome paradoxes and contradictions.

One of the earliest examples of the troublesome contradictions that the notion of truth can create is attributed to the ancient Greek philosopher Eubulides, who wrote, "A man says that he is not telling the truth. Is what he says true or false?" If what the man says is true, then the man is not telling the truth, so what he says must be false! But if what the man says is false, then it is false that he is not telling the truth, so what he says must be true! Thus, assuming what the man says is true leads us to a contradiction, and assuming what the man says is not true also leads us to a contradiction. In either case, the very notion of truth seems to generate a contradiction.

The same kinds of contradictions are generated by much simpler statements, such as "This statement is not true" or

> *The sentence in the box on this page is false.*

But why should it matter that the very concept of truth generates contradictions? Because, unfortunately, once a single contradiction is allowed, it is easy to prove with rigorous logic that any *statement whatsoever* is true. That is, anything can be proved once you accept a contradiction. This is fairly easy to show.

Let the letter *Q* stand for any statement you want, such as "Unicorns exist." Now suppose that you accept as true the statement "God is good." Call this statement *P*. And suppose you also accept as true the contradictory statement "God is not good." Call this statement *not-P*. Now consider the following statement:

(1) Either P is true or Q is true.

You must accept that statement 1 is true because you previously accepted that *P* is true. However, because you also accepted *not-P*, this means that *P* is not true. That is, you must also accept statement 2:

(2) P is not true.

Now you have accepted statements 1 and 2. But from statements 1 and 2, of course, it logically follows that

(3) Q is true.

And so you must accept that *Q* is true—that is, that unicorns exist! So by accepting the contradiction that *P* is true and that *not-P* is also true, we can logically prove that unicorns exist. In fact, anything at all can be proved with rigorous logic once a contradiction is accepted.

The terrible consequences that would follow should the concept of truth involve contradictions were what led twentieth-century logicians and mathematicians to invest considerable energy in trying to come up with ways to avoid contradictions. Unfortunately, this work has not yet come to any firm conclusions. The possibility that our notion of truth may be contradictory still lurks.

QUESTIONS

1. Can you conceive of some ways of avoiding the contradictions that truth seems to involve? Does Tarski's correspondence theory of truth suggest a way of avoiding these contradictions?
2. Can you conceive of some ways of avoiding the argument that once a contradiction is accepted, anything can be proved?